SCHOTTKY DIODE PARAMETERS EXTRACTION USING TWO DIFFERENT METHODS

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***Abstract*—In the present study, we determine exact analytical expression of the current flow through a Schottky barrier diode as a function of the input voltage. The Schottky diode is modeled by an electronic circuit containing four physical parameters: a series resistance R_s, a shunt resistance R_{sh}, a Schottky diode reverse saturation current I_s and a Schottky diode ideality factor η. Firstly, we solve the characteristic equation and determine the analytical expression of the input current I as a function of the input voltage of the Schottky diode V using the LambertW Function. Secondly, We present two different methods to extract the four physical parameters appearing in the electronic circuit. These methods are applied for two junctions: Iridium-Silicon Carbide Schottky barrier diode at $200K$ and Gold-Gallium Arsenide at $300K$. Finally, we compare the results obtained via the two methods presented.**

I. INTRODUCTION

In microelectronics, working with electronic devices requires precise knowledge of their physical parameters. In this work, the physical parameters of Schottky diode: the series resistance R_s, the shunt resistance R_{sh}, the Schottky diode reverse saturation current I_s and the Schottky diode ideality factor η are determined from its Current-Voltage characteristic. The extraction of the physical parameters is very important because it allows a better understanding of the carriers transport mechanisms and it helps in achieving technology advances. Until now, different methods of extraction have been developed in order to determine the parameters of Schottky diode. The simple one is the approximation method which requires the presence of linear region in the $ln(I)$ versus V plot [1]. In this method, only two parameters η and I_s can be obtained from the slope and projection of linear region and this relatively simple analysis fails when the series resistance R_s is large. Other methods which may be called two steps methods use some auxiliary functions and allow a preliminary determination of the series resistance R_s [2],[3]. The most recent published methods [4],[5] are the so-called lateral and vertical optimization methods. These methods use a minimization of the sum of squares of the relative differences between the measured and the fitting values of current. This optimization problem reduces to a resolution of a system of differential equations by Newton′s method. In this paper, the methods are based on the exact analytical solution of the current flow through a Schottky diode, where the current is given in term of the LambertW function. This function was introduced in the year 2000 by T.C. Banwell and A. Jayakumar when they proposed an exact analytical expression for the current flow in a diode driven by a voltage source through a series resistance [6]. In 2004, A. Jain and A. Kapoor have presented an exact closed form solution of the characteristic equation of a solar cell containing parasitic series and shunt resistances based on the LambertW function also [7]. In July 2005, M. Vargas-Drechsler has used Maple software [8],[9] and found the same exact analytical solution as T.C. Banwell and A. Jayakumar. Later, E. Assaid *et al.* have determined exact analytical expressions for the currents and the voltages of the circuit known as Graëtz bridge using LambertW function [10],[11]. In a previous work [12], we have used the solution given in [6] and have presented, for the first time, two different methods to determine the three physical parameters of a p-n junction .

In this study, we present two methods for the extraction of the Schottky diode physical parameters. The first method is based on the integration of the analytical expression of the input current $I = f(V)$. The second one uses a fit of the exact analytical expression of the input current to the experimental data.

II. BASIC EQUATIONS

The equation giving the current through a Schottky diode as a function of the input voltage writes:

$$I = I_s \left[exp\left(\frac{V - R_s I}{\eta V_{th}} \right) - 1 \right] + \frac{V - R_s I}{R_{sh}} \qquad (1)$$

Where $V_{th} = \dfrac{k_B T}{q}$ is the thermal voltage. q is the electron elementary charge, k_B is the Boltzmann constant and T is the absolute temperature.

Equation (1), which is transcendental in nature, is not solvable analytically in term of elementary functions. However, the explicit solution for the current can be expressed using the LambertW function.

After tedious calculations, equation (1) is expressed in the form:

$$we^w = x \tag{2}$$

Where:

$$w = \frac{R_s I - V}{\eta V_{th}} + \frac{R_s I_s + V}{\eta V_{th}(1 + G_p R_s)}, \tag{3}$$

and

$$x = \frac{R_s I_s}{\eta V_{th}(1 + G_p R_s)} \exp\left(\frac{R_s I_s + V}{\eta V_{th}(1 + G_p R_s)}\right) \tag{4}$$

The solution $w(x)$ of equation (2) is the multi-valued function $LambertW_k(x)$ [13]. In the present problem, the adequate branch of the LambertW function corresponds to $k = 0$ which is the only branch satisfying the condition $LambertW_0(x) = 0$ for $x = 0$. To simplify we put $LambertW = W$, the explicit solution of equation (1) writes:

$$I = \frac{\eta V_{th}}{R_s} W\left[\frac{R_s Is}{\eta V_{th}(1 + G_p R_s)} exp\left(\frac{R_s I_s + V}{\eta V_{th}(1 + G_p R_s)}\right)\right] + \frac{V}{R_s} - \frac{R_s I_s + V}{R_s(1 + G_p R_s)}, \tag{5}$$

where $G_p = \frac{1}{R_{sh}}$ is the shunt conductance.

Similarly, the explicit solution giving the input voltage in term of the input current can be expressed using the LambertW function as follows:

$$V = -\eta V_{th} W\left[\frac{I_s}{G_p \eta V_{th}} exp\left(\frac{(I + I_s)}{G_p \eta V_{th}}\right)\right] + I R_s + \frac{(I + I_s)}{G_p}. \tag{6}$$

III. PARAMETERS EXTRACTION METHODS

A. First Method

This method is inspired from the method proposed by Ortiz−Conde *et al.* for the determination of the five physical parameters of an illuminated real solar cell [14]. First of all, the analytical expression of the current flow through the Schottky diode is determined. To simplify, we put: $a = \frac{R_s I_s}{\eta V_{th}(1 + G_p R_s)}$, $b = R_s I_s$, $c = \eta V_{th}(1 + G_p R_s)$, $d = \frac{\eta V_{th}}{R_s}$, $f = \frac{-I_s}{(1 + G_p R_s)}$ and $g = \frac{G_p}{(1 + G_p R_s)}$. Then equation (5) becomes:

$$I = d\, W\left[a\, exp\left(\frac{V + b}{c}\right)\right] + gV + f \tag{7}$$

Next, we calculate $\int_0^V I\, dV$:

$$\int_0^V I\, dV = d\left[\frac{c}{2} W^2\left[a\, exp\left(\frac{x + b}{c}\right)\right] + c\, W^2\left[a\, exp\left(\frac{x + b}{c}\right)\right]\right]_0^V + [gx + f]_0^V \tag{8}$$

Equation (7) gives:

$$d\, W\left[a\, exp\left(\frac{V + b}{c}\right)\right] = I - gV - f \tag{9}$$

For $I = 0$ we have $V = 0$, then equation (9) becomes:

$$d\, W\left[a\, exp\left(\frac{b}{c}\right)\right] = -f \tag{10}$$

After tedious calculations, $\int_0^V I dV$ is expressed as:

$$\int_0^V I\, dV = c_{I2} I^2 + c_{V2} V^2 + c_{IV} IV + c_{I1} I + c_{V1} V \tag{11}$$

Where $c_{I2} = \frac{c}{2d}$, $c_{V2} = \frac{g}{2}(1 + \frac{cg}{d})$, $c_{IV} = \frac{-gc}{d}$, $c_{I1} = c(1 - \frac{f}{d})$ and $c_{V1} = \frac{gcf}{d} - gc + f$.

We have used Mathematica Software [15] to write a code where we calculate numerically $\int_0^V I\, dV$ from the experimental data given in reference [16]. Then we perform a two dimensional fitting of equation (11) to the numerical function $\int_0^V I\, dV$. The Schottky diode physical parameters: R_s, R_{sh}, I_s and η are determined from the coefficients c_{I2}, c_{V2}, c_{IV}, c_{I1} and c_{V1}: $R_s = \frac{-c_{IV}}{2c_{V2}}$, $R_{sh} = \frac{1}{2c_{V2}}$, $I_s = -c_{V1} - 2c_{I1}c_{V2} + c_{V1}c_{IV}$ and $\eta = \frac{c_{I1} - \frac{c_{V1}c_{VI}}{2c_{V2}}}{V_{th}}$.

Experimental data and optimized characteristics, obtained via the first method for both of the Schottky diodes, are plotted in figures 1 and 5. The deviation between the optimized characteristic and the corresponding experimental data for the two schottky diodes are drawn in figures 2 and 6.

B. Second Method

The second method was proposed for the first time by Jung and Guziewicz [16]. In this method, the Schottky diode parameters extraction was done using the FindFit function in the Mathematica Software package. The model equation is given by equation (5) and initial values of Schottky diode physical parameters must be introduced to achieve final parameters extraction.

In figures 3 and 7, the experimental data and the optimized characteristics obtained via the second method for both of the Schottky diodes are plotted. The deviation between the optimized characteristic and the corresponding experimental data for the two schottky diodes are drawn in figures 4 and 8.

IV. RESULTS

For the Ir−SiC junction

	Ir−SiC via the 1^{st} method	Ir−SiC via the 2^{nd} *method*
$R_s(\Omega)$	3.48	3.479
$R_{sh}(\Omega)$	$3.105\ 10^9$	$3.105\ 10^9$
$I_s(A)$	$1.919\ 10^{-20}$	$1.919\ 10^{-20}$
η	1.0179	1.0180

For the Au−GaAs junction

	Au−GaAs via the 1^{st} method	Au−GaAs via the 2^{st} method
$R_s(\Omega)$	717.99	718
$R_{sh}(\Omega)$	2.207 10^{10}	2.207 10^{10}
$I_s(A)$	4.099 10^{-17}	4.09 10^{-17}
η	1.0539	1.0539

V. Conclusion

In this study, we have presented two different methods for the extraction of a Schottky barrier diode physical parameters. The Schottky barrier diode was modeled by an electronic circuit containing parasitic series and shunt resistances and a non ideal diode with a reverse saturation current and an ideality factor. The first method is based on the analytical and numerical calculation of the area under the characteristic $I = f(V)$. In the second method we use the FindFit function of the Mathematica software package and fit the experimental data to the analytical expression of the Schottky diode input current. A comparative study of the current deviation between the optimized results and the experimental data shows that the first method leads to best results for the Au−GaAs junction. Nevertheless, for the Ir−SiC junction the best results are obtained by the second method. At the end, we must point out that the first method presented does not require any introduction of initial values, contrary to the second method where the knowledge of initial values is obligatory.

Acknowledgment

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Graphs

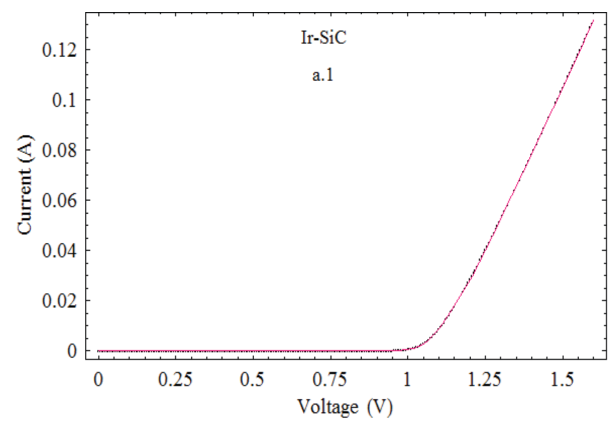

Fig. 1. Experimental data (dots) and numerical characteristic (line) obtained via the first method

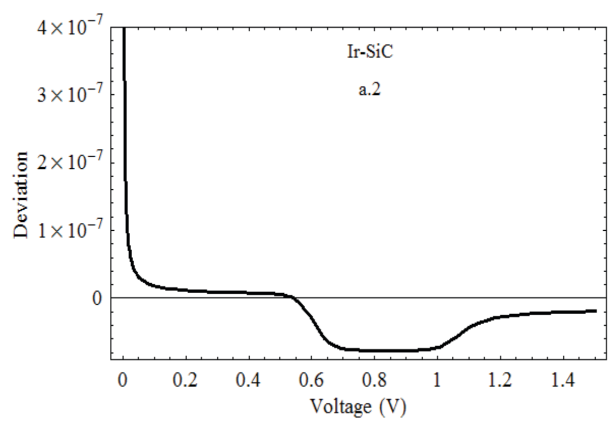

Fig. 2. Current deviation between optimized characteristic obtained via the first method and experimental data

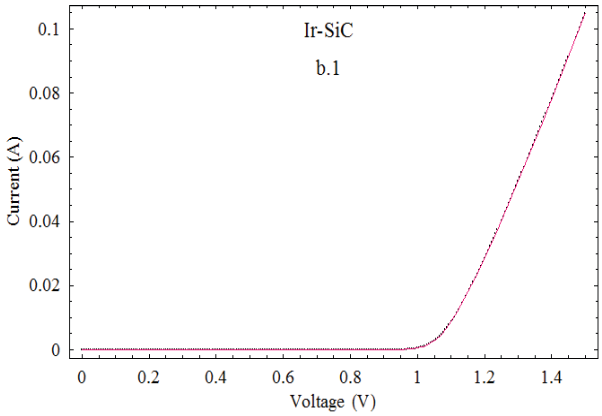

Fig. 3. Experimental data (dots) and numerical characteristic (line) obtained via the second method

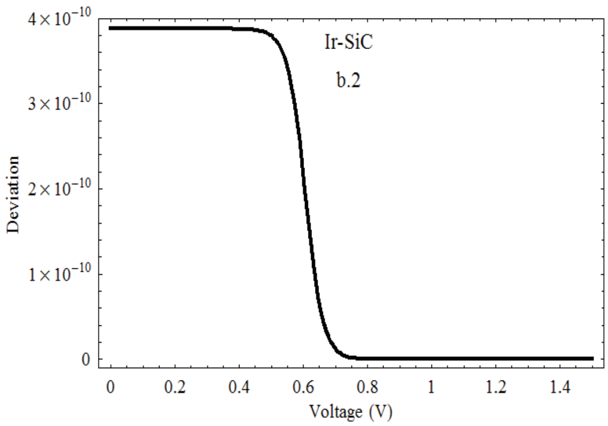

Fig. 4. Current deviation between optimized characteristic obtained via the second method and experimental data

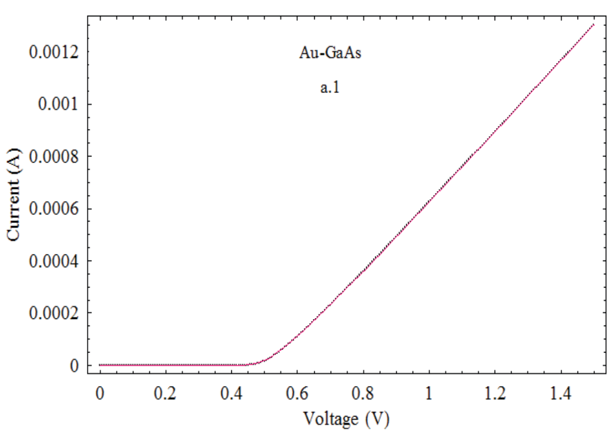

Fig. 5. Experimental data (dots) and numerical characteristic (line) obtained via the first method

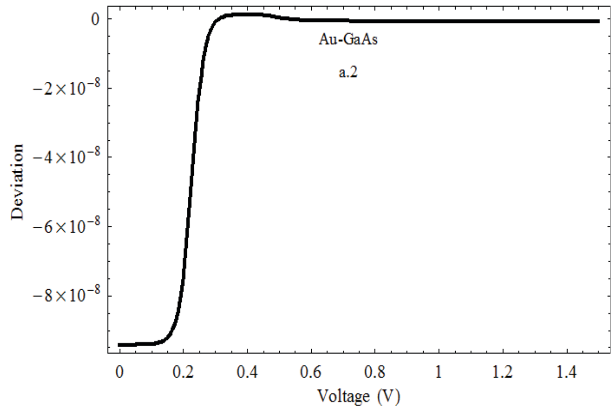

Fig. 6. Current deviation between optimized characteristic obtained via the first method and experimental data

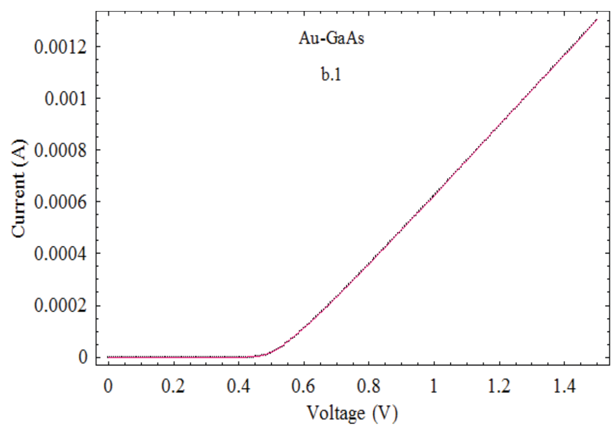

Fig. 7. Experimental data (dots) and numerical characteristic (line) obtained via the second method

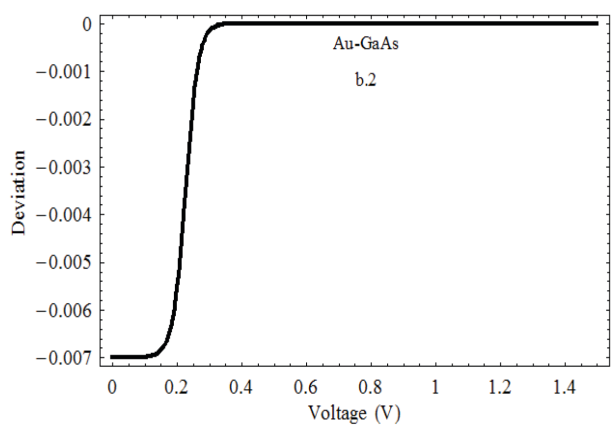

Fig. 8. Current deviation between optimized characteristic obtained via the second method and experimental data